

Conditional probability

Conditional or posterior probabilities

e.g., $P(\text{cavity}|\text{toothache}) = 0.8$

i.e., **given that *toothache* is all I know**

NOT “if *toothache* then 80% chance of *cavity*”

(Notation for conditional distributions:

$\mathbf{P}(\text{Cavity}|\text{Toothache}) = 2\text{-element vector of } 2\text{-element vectors})$

If we know more, e.g., *cavity* is also given, then we have

$P(\text{cavity}|\text{toothache}, \text{cavity}) = 1$

Note: the less specific belief **remains valid** after more evidence arrives, but is not always **useful**

New evidence may be irrelevant, allowing simplification, e.g.,

$P(\text{cavity}|\text{toothache}, 49ersWin) = P(\text{cavity}|\text{toothache}) = 0.8$

This kind of inference, sanctioned by domain knowledge, is crucial

Conditional probability

Definition of conditional probability:

$$P(a|b) = \frac{P(a \wedge b)}{P(b)} \text{ if } P(b) \neq 0$$

Product rule gives an alternative formulation:

$$P(a \wedge b) = P(a|b)P(b) = P(b|a)P(a)$$

A general version holds for whole distributions, e.g.,

$$\mathbf{P}(\textit{Weather}, \textit{Cavity}) = \mathbf{P}(\textit{Weather}|\textit{Cavity})\mathbf{P}(\textit{Cavity})$$

(View as a 4×2 set of equations, **not** matrix mult.)

Chain rule is derived by successive application of product rule:

$$\begin{aligned} \mathbf{P}(X_1, \dots, X_n) &= \mathbf{P}(X_1, \dots, X_{n-1}) \mathbf{P}(X_n | X_1, \dots, X_{n-1}) \\ &= \mathbf{P}(X_1, \dots, X_{n-2}) \mathbf{P}(X_{n-1} | X_1, \dots, X_{n-2}) \mathbf{P}(X_n | X_1, \dots, X_{n-1}) \\ &= \dots \\ &= \prod_{i=1}^n \mathbf{P}(X_i | X_1, \dots, X_{i-1}) \end{aligned}$$

Inference by enumeration

Start with the joint distribution:

	<i>toothache</i>		\neg <i>toothache</i>	
	<i>catch</i>	\neg <i>catch</i>	<i>catch</i>	\neg <i>catch</i>
<i>cavity</i>	.108	.012	.072	.008
\neg <i>cavity</i>	.016	.064	.144	.576

For any proposition ϕ , sum the atomic events where it is true:

$$P(\phi) = \sum_{\omega: \omega \models \phi} P(\omega)$$

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$$P(\text{cavity} \vee \text{toothache}) = 0.108 + 0.012 + 0.072 + 0.008 + 0.016 + 0.064 = 0.28$$

Inference by enumeration

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	<i>catch</i>	\neg <i>catch</i>	<i>catch</i>	\neg <i>catch</i>
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Can also compute conditional probabilities:

$$\begin{aligned} P(\neg \text{cavity} | \text{toothache}) &= \frac{P(\neg \text{cavity} \wedge \text{toothache})}{P(\text{toothache})} \\ &= \frac{0.016 + 0.064}{0.108 + 0.012 + 0.016 + 0.064} = 0.4 \end{aligned}$$