



# Artificial Intelligence

*CSC348 Unit 3: Problem Solving and Search*  
**Part 2: Informed Search Techniques**

*Syedur Rahman*  
*Lecturer, CSE Department*  
*North South University*  
*[syedur.rahman@wolfson.oxon.org](mailto:syedur.rahman@wolfson.oxon.org)*

# *Artificial Intelligence: Lecture Notes*

The lecture notes from the introductory lecture and this unit will be available shortly from the following URL:

- <http://www.geocities.com/syedatnsu/>

## **Acknowledgements**

- These lecture notes contain material from the following sources
  - *Logical Programming and Artificial Intelligence* by S. Kapetanakis, 2004
  - *Artificial Intelligence: A modern approach* by S. Russell and P. Norvig, International Edition, 2nd edition
  - *Intelligent Systems* by S. Clark, 2005

# *Best-first search*

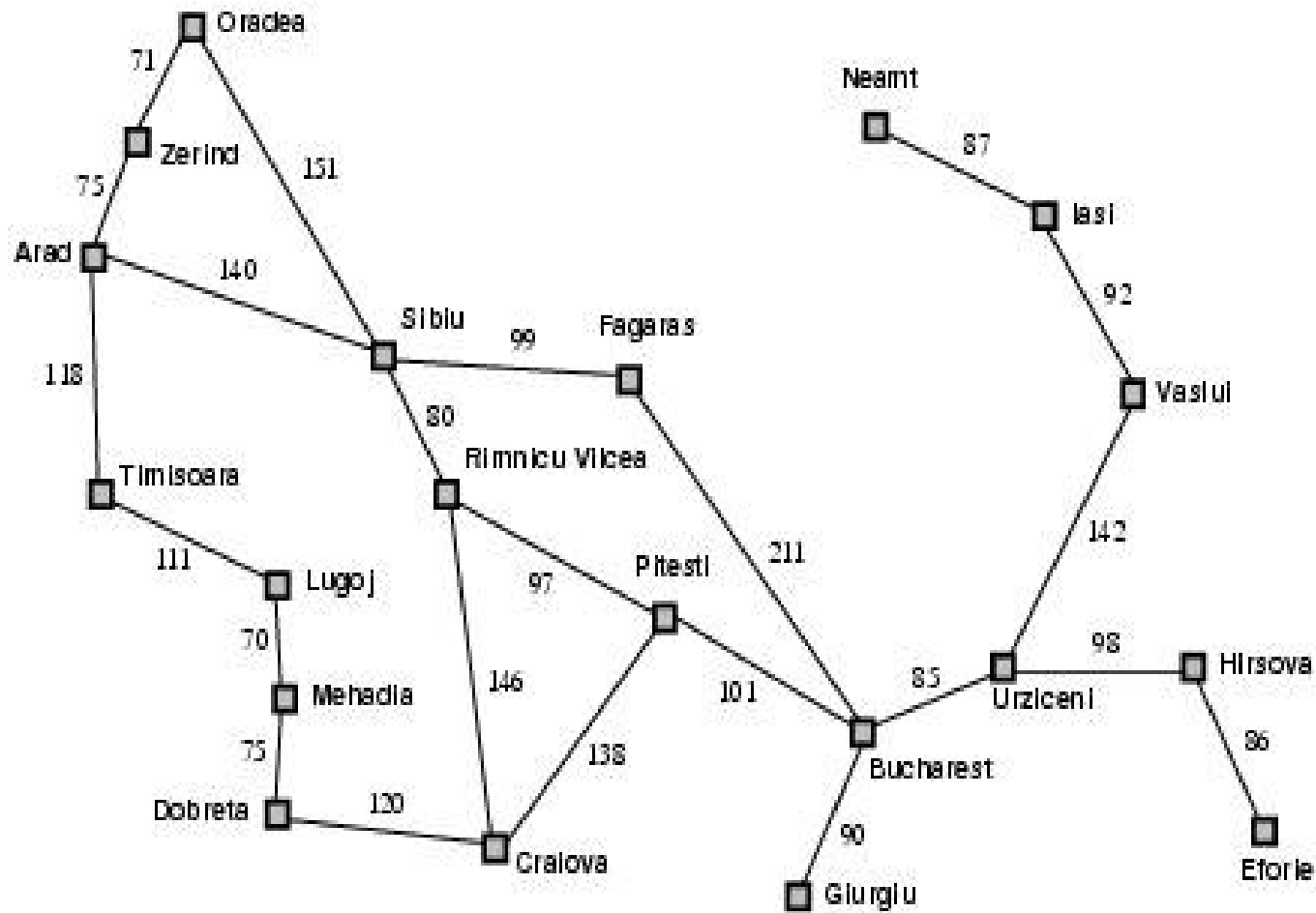
- Node is selected for expansion based on an *evaluation function  $f(n)$*
- Evaluation function estimates distance to the goal
- Choose node which *appears* best
- Implementation:
  - fringe is a priority queue sorted in ascending order of  $f$ -values



# *A heuristic function $h(n)$*

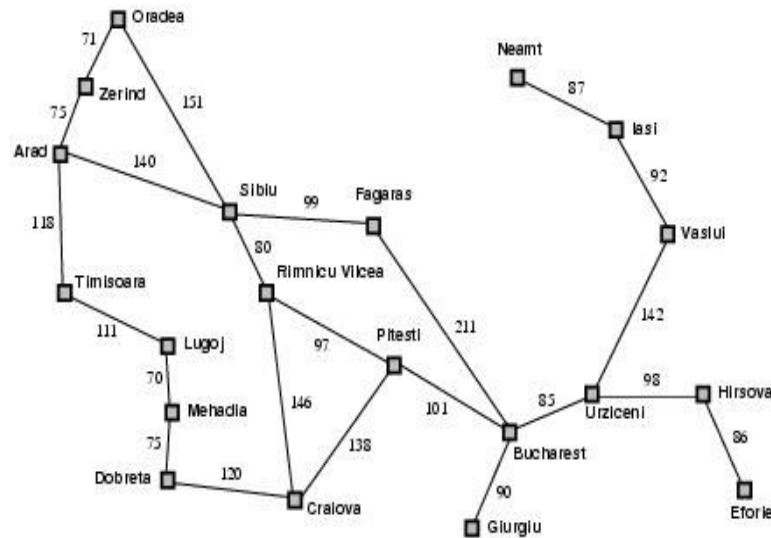
- Dictionary defn: *“A rule of thumb, simplification, or educated guess that reduces or limits the search for solutions in domains that are difficult and poorly understood”*
- For best-first search:  
 $h(n)$  = estimated cost of the cheapest path from node  $n$  to goal node

# *Example: Romania*



# Romania with step costs in km

Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Dobreta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374



- $h_{SLD}$  = straight-line distance heuristic
- $h_{SLD}$  cannot be computed from the problem description itself
- In **greedy best-first search**  $f(n) = h(n)$ 
  - Expand node that is closest to goal

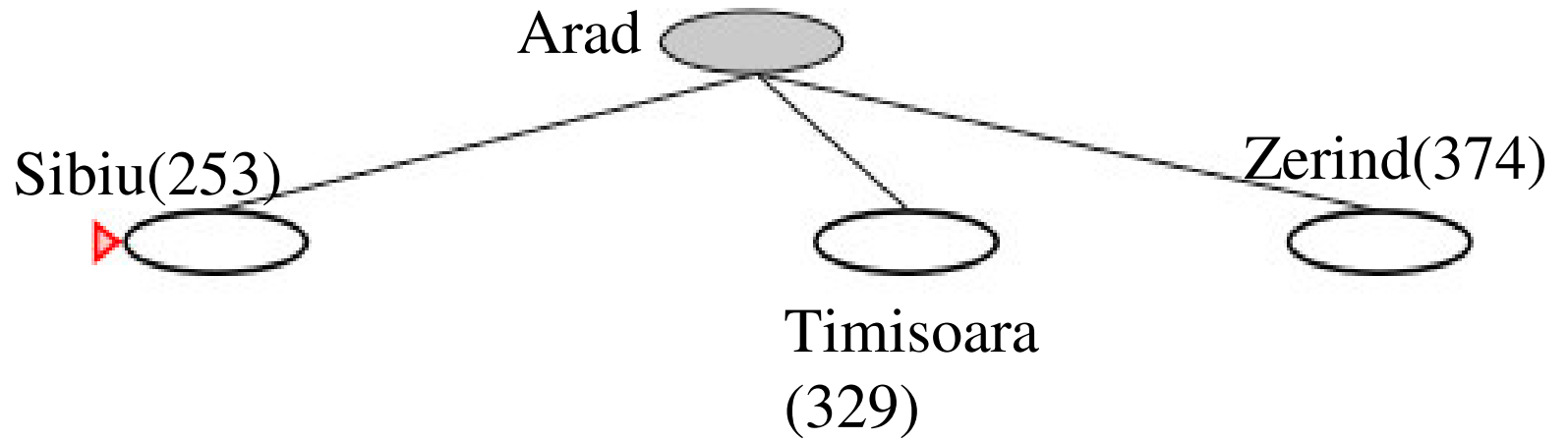
# *Greedy search example*

Arad (366)



- Greedy search to solve the Arad to Bucharest problem

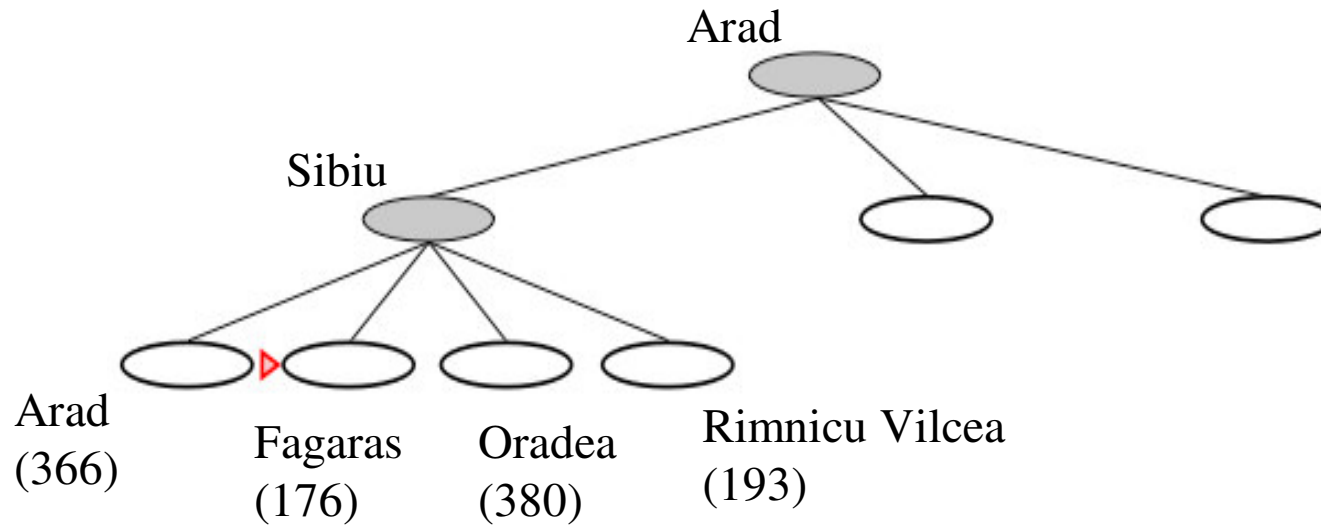
# *Greedy search example*



- Greedy best-first search will select Sibiu

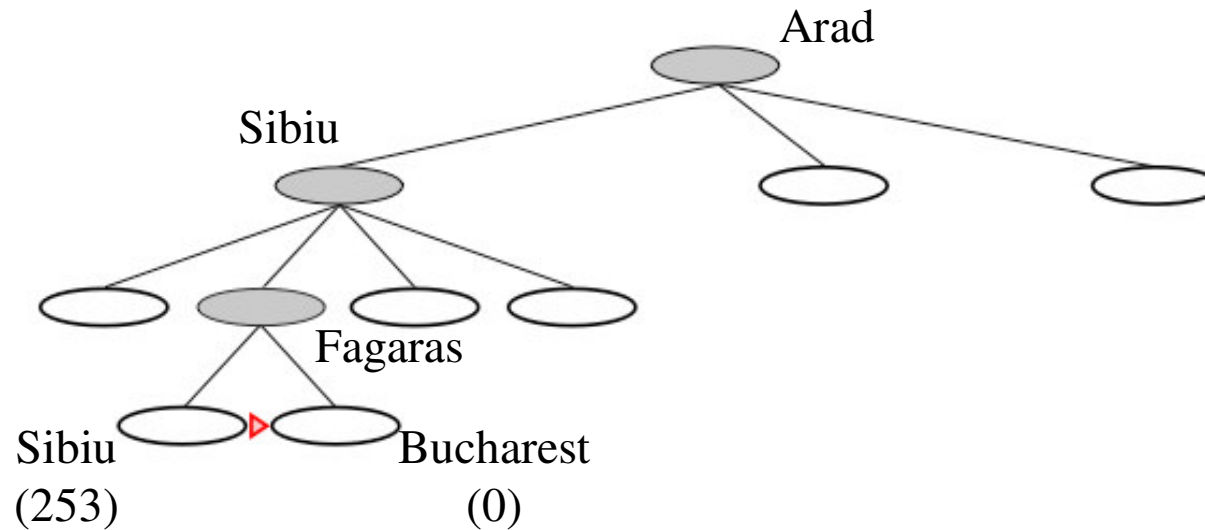


# *Greedy search example*



- Greedy best-first search will select Fagaras

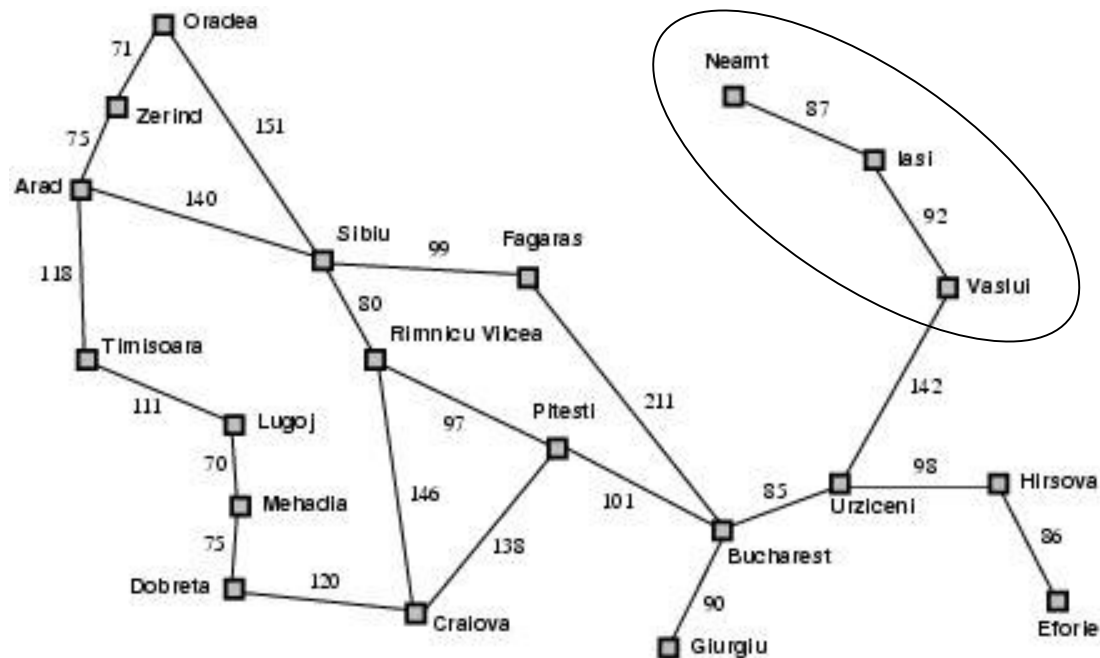
# Greedy search example



- Goal reached
  - For this example no node is expanded that is not on the solution path
  - But not optimal (see Arad, Sibiu, Rimnicu Vilcea, Pitesti)

# Greedy search: evaluation

- Complete or optimal: no
  - Minimizing  $h(n)$  can result in false starts, e.g. Iasi to Fagaras
  - Check on repeated states



# *Greedy search: evaluation*

- Time and space complexity:
  - In the worst case all the nodes in the search tree are generated:  $O(b^m)$   
(m is maximum depth of search tree and b is branching factor)
  - But: choice of a good heuristic can give dramatic improvement

# *A\** search



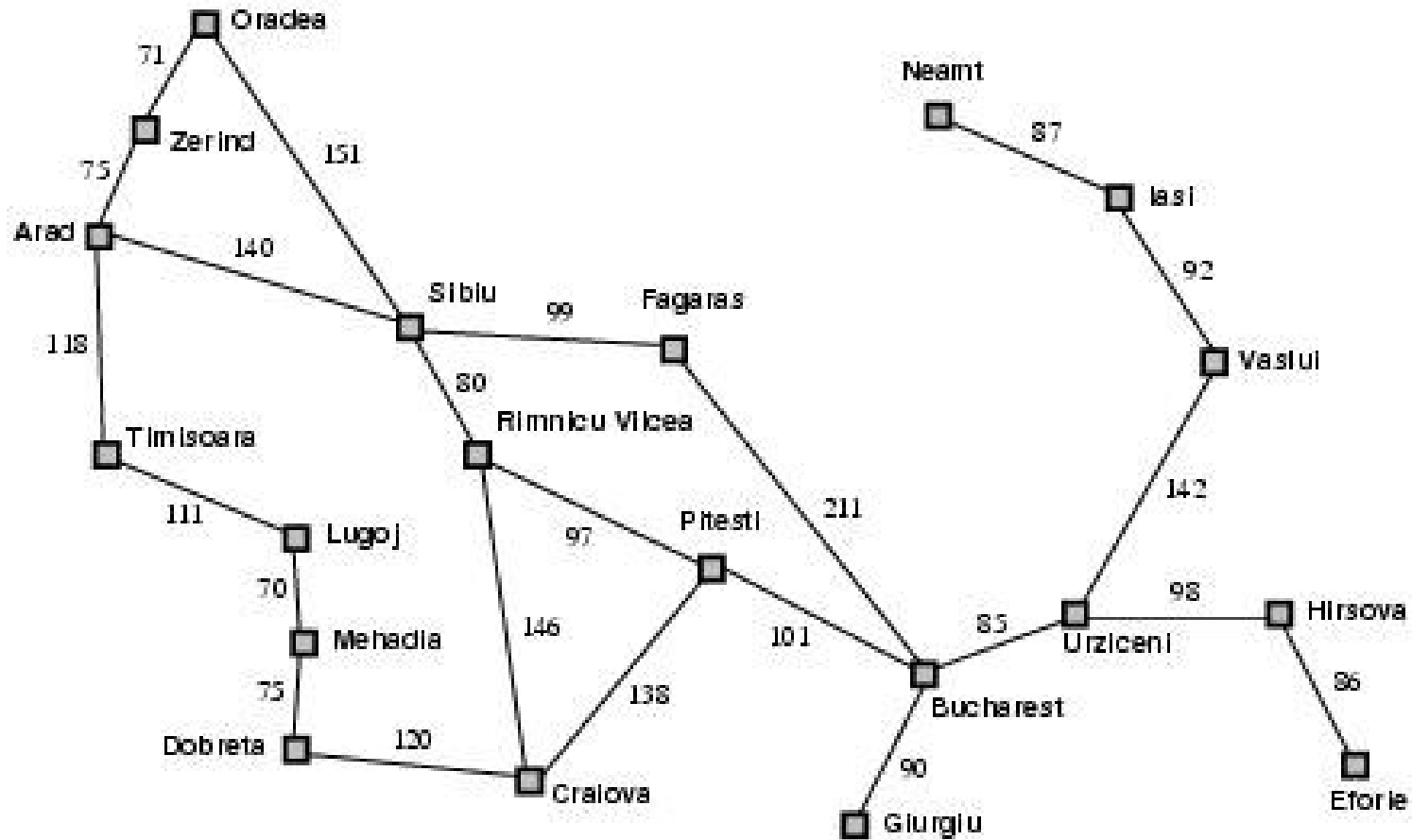
- Best-known form of best-first search
- Idea: avoid expanding paths that are already expensive
- Evaluation function  $f(n) = g(n) + h(n)$ 
  - $g(n)$ : the cost (so far) to reach the node
  - $h(n)$ : estimated cost to get from the node to the goal
  - $f(n)$ : estimated total cost of path through  $n$  to goal
- A\* search is both complete and optimal if  $h(n)$  satisfies certain conditions



# *A\** search

- A\* search is optimal if  $h(n)$  is an **admissible heuristic**
- A heuristic is admissible if it *never overestimates* the cost to reach the goal
  - $h(n) \leq h^*(n)$  where  $h^*(n)$  is the true cost from  $n$
- Admissible heuristics are optimistic about the cost of solving the problem
- e.g.  $h_{SLD}(n)$  never overestimates the actual road distance

# Romania example



# *A\* search example*

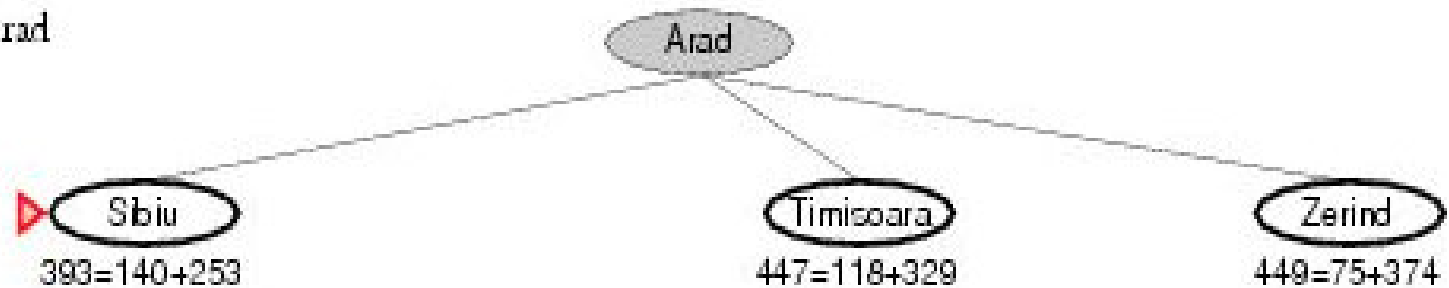
(a) The initial state





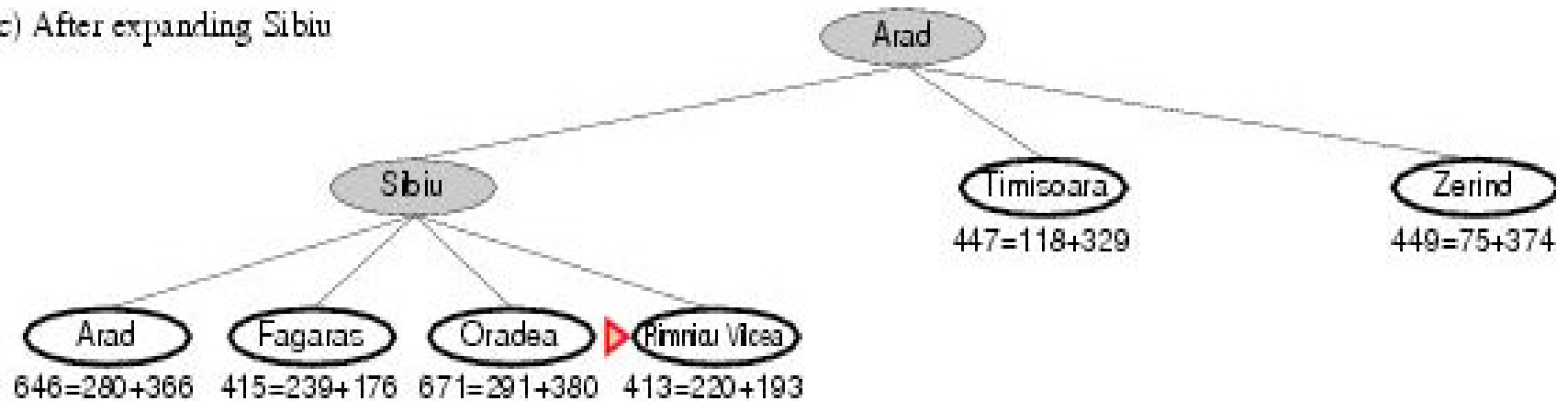
# *A\* search example*

After expanding Arad



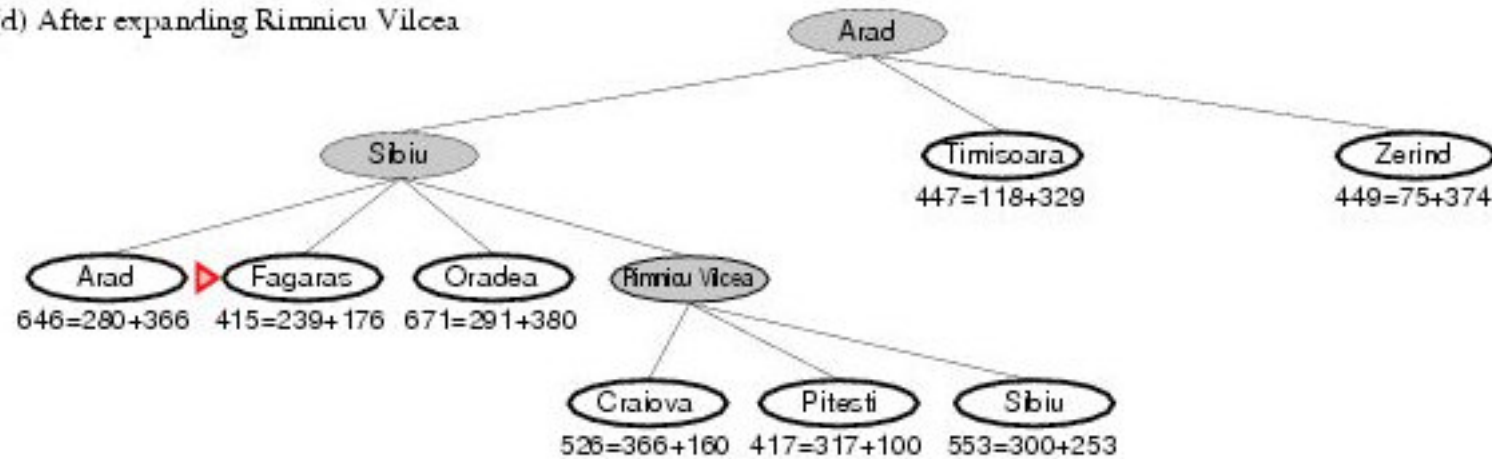
# *A\* search example*

(c) After expanding Sibiu



# *A\* search example*

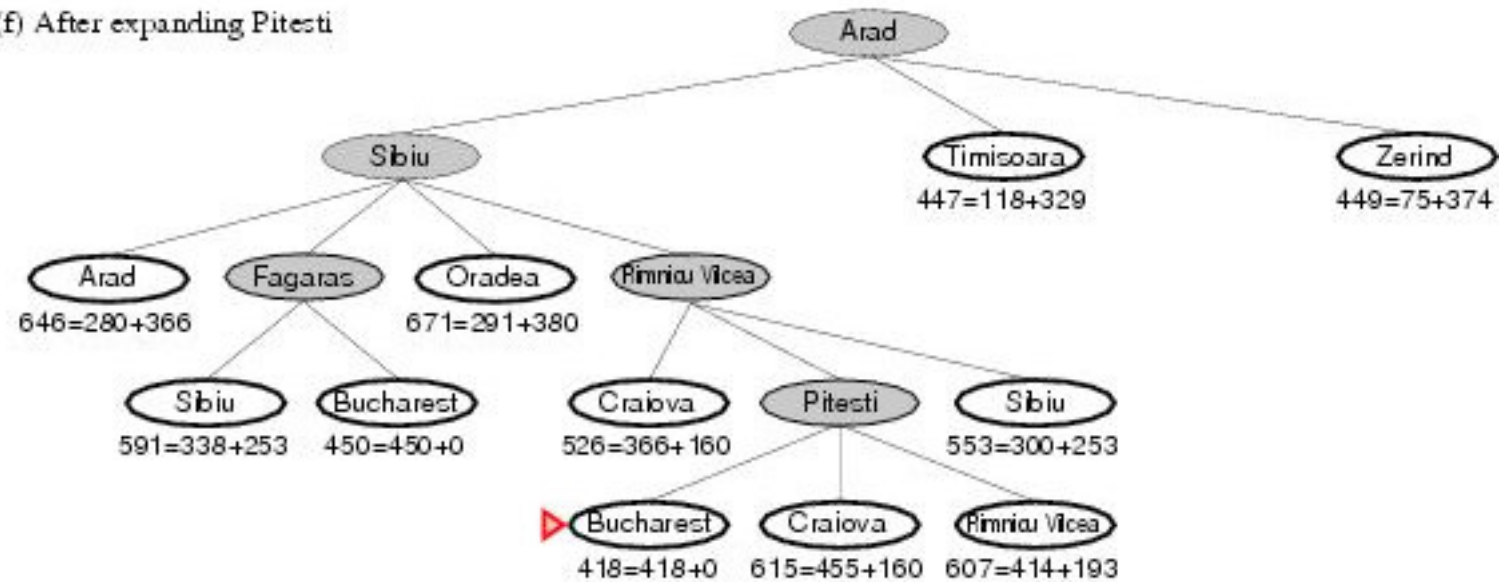
(d) After expanding Rimnicu Vilcea





# A\* search example

(f) After expanding Pitesti



# *A\** and GRAPH-SEARCH

- GRAPH-SEARCH discards new paths to a repeated state
  - So may discard the optimal path
- Solutions:
  - Remove more expensive of the two paths
    - But requires extra book-keeping
  - Ensure that the optimal path to any repeated state is always the first one followed
    - Requires extra condition on  $h(n)$ : **consistency** (or **monotonicity**)

# Consistency

- A heuristic is consistent if

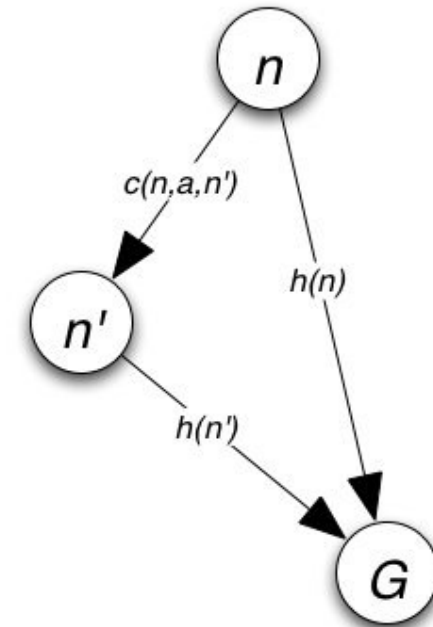
$$h(n) \leq c(n, a, n') + h(n')$$

- If  $h$  is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &\geq f(n) \end{aligned}$$

- i.e.  $f(n)$  is nondecreasing along any path

- and so  $A^*$  using GRAPH-SEARCH expands nodes in non-decreasing order of  $f(n)$



# *A\* search: evaluation*



- Complete: yes
  - Unless there are infinitely many nodes with  $f < f(G)$
  - Since bands of increasing  $f$  are added
- Optimal: yes
  - A\* is **optimally efficient** for any given  $h(n)$ : no other optimal algorithm is guaranteed to expand fewer nodes



# *A\* search: evaluation*



- Time complexity:
  - number of nodes expanded is still exponential in length of solution
- Space complexity:
  - All generated nodes are kept in memory
  - A\* usually runs out of space before running out of time